What We Will Cover in This Section

- Introduction to Multivariate Statistics.
- Multivariate Analysis of Variance, Overview.
- MANOVA, Example.
- Multivariate Analysis of Covariance (MANCOVA).
- Key Learning Principles.

A Problem

A group of students wanted to know if there were significant differences in performance between a group of dyslexic and non-dyslexic students in a simple visual-motor skills task. Volunteers were asked to perform on a mirror-tracing apparatus. The number of errors and time to trace the design was recorded for each person.
Multivariate Statistics

• In experimental research.
  – Inferential statistical techniques used to assess the differences between means when there is more than one dependent variable.

• In correlational research.
  – Inferential statistical techniques used to assess the degree of association between more than two variables.

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t-test vs. ANOVA vs. MANOVA

<table>
<thead>
<tr>
<th>Test</th>
<th>Number of independent variables</th>
<th>Number of dependent variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-test</td>
<td>One</td>
<td>One</td>
</tr>
<tr>
<td>ANOVA</td>
<td>Multiple</td>
<td>One</td>
</tr>
<tr>
<td>MANOVA</td>
<td>Multiple</td>
<td>Multiple</td>
</tr>
</tbody>
</table>

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Correlational Examples

<table>
<thead>
<tr>
<th>Test</th>
<th>Number of predictors</th>
<th>Number of criterion variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Correlation</td>
<td>One</td>
<td>One</td>
</tr>
<tr>
<td>Multiple Regression</td>
<td>Multiple</td>
<td>One</td>
</tr>
<tr>
<td>Canonical Correlation</td>
<td>Multiple</td>
<td>Multiple</td>
</tr>
</tbody>
</table>
Multivariate Analysis of Variance: MANOVA

Why Have Several DVs?
1. From a validity point of view, multiple dependent measures are better than a single dependent measure.
2. A strong treatment will affect people in many ways.
3. It is more efficient to do one study assessing the impact of the independent variable(s) three things than three studies assessing their impact on one thing.

Benefits of MANOVA
1. Opportunity to use multiple measures.
2. May not find differences between two correlated dependent variables if assessed in separate studies.
3. Multiple ANOVAs will lead to inflated Type I error rate.
Concept: How it is Done

- Simple MANOVA develops a linear combination of the dependent variables.
  - This equation has the following format.
  \[ V_{new} = a_1 V_1 + a_2 V_2 + a_3 V_3 + \ldots + a_n V_n \]
  - This combination takes into account any possible correlation between the variables.
  - This equation is designed in such a way as to provide maximum differences between the treatment groups on \( V_{new} \).

MANOVA Null Hypothesis

\[
\begin{align*}
\text{ANOVA} & & H_o : \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k \\
\text{MANOVA} & & H_o : \mu_{V_1} = \mu_{V_2} = \mu_{V_3} = \ldots = \mu_{V_k}
\end{align*}
\]

Usual Statistical Test

\[
\begin{align*}
\text{ANOVA F Test} & & F = \frac{SS_{\text{Between}}}{SS_{\text{Within}}} \quad \text{Larger indicates larger differences.} \\
\text{MANOVA Wilks Lambda} & & \Lambda = \frac{|W|}{|T|} \quad \text{SMALLER indicates larger variability between the vector of means.}
\end{align*}
\]
Conduct MANOVA

Significant? No → Stop

Conduct ANOVAs for each variable to identify distinguishing variables.

Significant? No → Stop

Conduct post hoc tests to identify different groups.

Correcting the Alpha Level

Conducting multiple ANOVAs increases the probability of a Type 1 Error.

To correct divide $\alpha$ by the number of dependent variables. This equalizes the probability across the variables.

This is called the Bonferroni adjustment.

Bonferroni Example

If there are two dependent variables, then the Bonferroni corrected level would be $.05/2$ or $.025$. 
**MANOVA Assumptions**

1. The observations within each sample are random and independent of each other.
2. The dependent variables are multivariate normal.
3. The covariance matrices for the dependent variables are equal.
4. The relationship between the dependent variables is linear.

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**Homogeneity of Variance**

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**Total: Sum of Squares, Sum of Cross Products**

<table>
<thead>
<tr>
<th></th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>SS_{V1}</td>
<td>SCP_{V1xV2}</td>
<td>SCP_{V1xV3}</td>
</tr>
<tr>
<td>V2</td>
<td>SCP_{V1xV2}</td>
<td>SS_{V2}</td>
<td>SCP_{V2xV3}</td>
</tr>
<tr>
<td>V3</td>
<td>SCP_{V1xV3}</td>
<td>SCP_{V2xV3}</td>
<td>SS_{V3}</td>
</tr>
</tbody>
</table>
Comparing: ANOVA vs. MANOVA

ANOVA: \[ SS_{Total} = SS_{Between} + SS_{Within} \]

MANOVA: \[ SSCP_{Total} = SSCP_{Between} + SSCP_{Within} \]

\[ T = B + W \]

MANOVA Computation

\[ \Lambda = \left| \frac{W}{T} \right| = \left| \frac{W}{B + W} \right| \]

\[ \eta^2 = 1 - \Lambda \]

Example

A researcher was interested in the possibility that a new type of therapy would influence the cognitive levels of people with disabilities. To control for severity of the disability she divided the group into three levels of disability: Mild, Moderate, and Severe. The dependent variable in this study was the influence of the therapy on three cognitive skills tests: WRAT-R, WRAT-A, and IQ.
Step 1. MANOVA

<table>
<thead>
<tr>
<th>Source</th>
<th>Wilks' Lambda</th>
<th>Df₁</th>
<th>Df₂</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>.13772</td>
<td>2</td>
<td>11</td>
<td>34.43**</td>
</tr>
<tr>
<td>Disability</td>
<td>.25526</td>
<td>4</td>
<td>22</td>
<td>5.38*</td>
</tr>
<tr>
<td>T X D</td>
<td>.90807</td>
<td>4</td>
<td>22</td>
<td>.27</td>
</tr>
</tbody>
</table>

*p< .05, **p<.01

Step 2. ANOVA

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>2090.88</td>
<td>1</td>
<td>2090.88</td>
<td>46.1</td>
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<tr>
<td>WRAT-A</td>
<td>1494.22</td>
<td>1</td>
<td>1494.22</td>
<td>33.2</td>
</tr>
<tr>
<td>Disability</td>
<td>WRAT-R</td>
<td>520.77</td>
<td>2</td>
<td>260.38</td>
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<tr>
<td>WRAT-A</td>
<td>1126.77</td>
<td>2</td>
<td>563.38</td>
<td>12.5</td>
</tr>
<tr>
<td>T X D</td>
<td>WRAT-R</td>
<td>2.11</td>
<td>2</td>
<td>1.056</td>
</tr>
<tr>
<td>WRAT-A</td>
<td>52.78</td>
<td>2</td>
<td>26.38</td>
<td>.587</td>
</tr>
</tbody>
</table>

Error: WRAT-R 544.00 | 12 | 45.33 |
WRAT-A 539.33 | 12 | 44.94 |
Step 3. Post hoc Tests

Too damn many to show you.

THE END