Comparing Two Means: Z-test

What We Will Cover in This Section

- Introduction.
- The Z-test review.
- Other problems.

The Z-Test
Application: The Z-test

The average age of registered voters in Slippery Gulch is $\mu = 39.7$ years old and the standard deviation, $\sigma$, is 10.

The League of Women Voters wanted to encourage younger people to vote so they sponsored a series of educational articles and television commercials on the benefits of voting.

Afterwards, a sample of 12 voters at the latest election was found to have a mean age of 28.2 years.

Did the advertising have an effect on voters or could this result have been a result of random error?

The Statistical Model, Again

Sample Mean 28.2

Population $\mu$ 39.7

Is this sample mean so far away from the population mean that we should conclude that it does not represent this population?

How to Think About This

Could a sample with a mean of 28.2 have occurred in a distribution where the mean is 39.7 and the standard deviation is 10?

or

Does the sample with $M = 28.2$ represent a different population?

What distinguishes this ‘different’ population would be the commercials.
Decision Issues

• How do you determine far away?
  – What measure do we have to determine how far away a sample mean is from the population mean?
• How do we determine if this mean is rare?
  – What is rare?

The Z-Test Formula

\[ Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \]

How to Compute, Step #1

1. Determine the critical value for a one-tail test where \( p < .05 \).

   Critical Value = -1.64
2. Calculate the standard error.

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} \]

\[ \sigma_{\bar{x}} = \frac{10}{\sqrt{12}} \]

\[ \sigma_{\bar{x}} = 3.464 \]

\[ \sigma_{\bar{x}} = 2.89 \]

3. Calculate how far the sample mean is from the population mean in SE units.

\[ Z = \frac{\bar{X} - \mu}{\sigma_{\bar{x}}} \]

\[ Z = \frac{28.2 - 39.7}{3.464} \]

\[ Z = -3.98 \]

4. Compare the Z-score to the critical value.

Population \( \mu \)

Sample Mean

28.2, \( Z = -3.98 \)

Critical value, \( p<.05 \) = 1.64
Properties of the Z-test

• What you can learn. Does a sample mean (M) differ significantly from a population mean (μ) or could this difference have occurred by chance.

• Assumptions.
  – Interval or ratio scales.
  – Know μ and σ.
  – Know the sample mean.
  – Know the sample size.

ALPHA Level (α)

• ALPHa is the statistical statement of something that is rare.
  – Traditionally, alpha is defined as something that would happen 5% of the time or less.
  – This is shown by: p < .05.

Critical Values for α

<table>
<thead>
<tr>
<th>Critical Value</th>
<th>Type of test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One tail</td>
</tr>
<tr>
<td>.05</td>
<td>1.64</td>
</tr>
<tr>
<td>.01</td>
<td>2.33</td>
</tr>
</tbody>
</table>
Example #2

Melody Tunne thought that listening to music while taking a statistics test would either be relaxing, increasing performance, or distracting, decreasing performance. She did not know which.

1. Is this a one-tail or two-tail test?
2. What alpha level should Melody set?

Melody’s Data

- The mean for the population of students who have taken the statistics test is $\mu = 50$.
- The standard deviation for all students is $\sigma = 12$.
- Melody got a sample of 49 students who listened to music while taking the test.
  - Their mean was 54.63
  - Their standard deviation was 7.

The END?