Graduate Statistics

The t-test

What We Will Cover in This Section

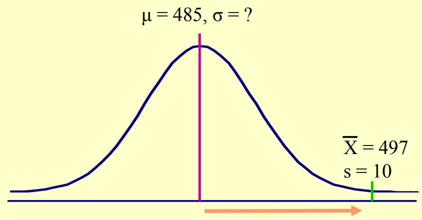
- Introduction
- One-sample t-test.
- Independent samples t-test.
- Dependent samples t-test.
- Power and effect size.
- Key learning points.

A Research Situation

A high school wants to know if a special SAT preparation program has influenced the scores of a group of 25 participants.Historically the mean verbal score for all of their graduating seniors is μ = 485, but they don't have the standard deviation. The sample has a mean SAT score of 497 with a standard deviation of 10.

1. What is the research hypothesis?
2. What is H_o?
3. What is the statistical hypothesis?
4. Is this a one-tailed or two-tailed test?

Draw The Picture (DTP)



z-test and the Single Sample t-test

Known statistics	z-test	Single sample t-test
μ	Yes	Yes
σ	Yes	**No**
M	Yes	Yes
s	Yes*	Yes

Comparing the Formulas

	z- test	Single Sample t-test
Standard Error of the Mean	$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$	$\sigma_{\overline{X}} = \frac{s}{\sqrt{N}}$
Test	$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$	$t_{(N-1)} = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$

Degrees of Freedom (df)

- Developed from the notion that when you know that a group of N numbers sum to S, and if you know N-1 of the numbers, the N^{th} number is fixed.
- Example.
  If a group of 4 numbers add up to 15 and three of the numbers are 5, 6, and 2, what is the fourth number?

In this case you have N-1 degrees of freedom.

Back to the Example: Computation

A high school wants to know if a special SAT preparation program has influenced the scores of a group of **25 participants.** Historically the mean verbal score for all of their graduating seniors **is μ = 485**, but they don't have the standard deviation. The sample has a **mean SAT score of 497** with a **standard deviation of 10.**

$$\sigma_{\overline{X}} = \frac{10}{\sqrt{25}}$$

$$\sigma_{\overline{X}} = 2$$

$$t_{(N-1)} = \frac{497 - 485}{2}$$

$$t_{(24)} = 6.00$$

α, 1 tail → PROPORTION IN ONE TAIL

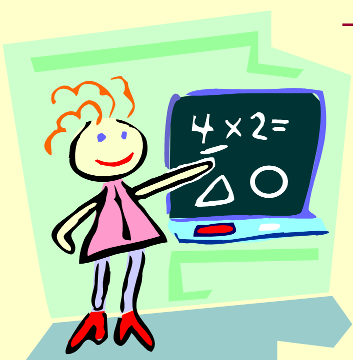
α, 2 tail → PROPORTION IN TWO TAILS

	PROPORTION IN ONE TAIL					
	0.25	0.10	0.05	0.025	0.01	0.005
	PROPORTION IN TWO TAILS					
df	0.50	0.20	0.10	0.05	0.02	0.01
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.658	1.980	2.358	2.617
∞	0.674	1.282	1.645	1.960	2.326	2.576

Table III of R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research,*

Assumptions of Single Sample t-test

1. The population mean is available.
2. The population distribution is normal.
3. The observations are *independent*.
4. Measurement is done on an interval or ratio scale.



Independent Sample t-test

Independent Sample t-Test

The grades for a group of CUP soccer players tend to be somewhat below average. This might be a result of bouncing balls off their heads. To deal with this, twenty first-year players are equipped with helmets. A control group of 20 players played without helmets. At the end of the school year their grades are compared.

1. What is the research hypothesis?
2. What is H_o?
3. What is the statistical hypothesis?
4. Is this a one-tailed or two-tailed test?

Model

$$t_{(df)} = \frac{\overline{X}_1 - \overline{X}_2}{\text{standard error}}$$

Single Sample vs. Independent Sample t-test

Known statistics	Single Sample t	Independent Sample t
μ	Yes	No
σ	No	No
M_1	Yes	Yes
s_1	Yes	Yes
M_2		Yes
s_2		Yes

Problem!

How do you get the Standard Error? →

$$\frac{\bar{X}_1 - \bar{X}_2}{\sigma_{\bar{X}}}$$

First: Pooled Error Variance (s^2_p)

- Represents the mean variance estimate from the two samples.
- Weighted by the sample size.
- Two estimates mean *df* = (N_1+N_2-2).

$$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$$

Second: Standard Error of Difference Between Two Means

- Represents the *Standard Error* when sampling the difference between two means.

$$S_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{S_P^2}{N_1} + \frac{S_P^2}{N_2}}$$

Third: Compute t

$$t_{(N_1 + N_2 - 2)} = \frac{\overline{X}_1 - \overline{X}_2}{S_{\overline{X}_1 - \overline{X}_2}}$$

Total degrees of freedom

Standard Error of the Difference

Soccer Study Results, Step 1

Helmet	No-Helmet
M = 2.50	M = 1.90
SS = 4.75	SS = 6.84
N = 20	N = 20

$$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$$

$$s_p^2 = \frac{4.75 + 6.84}{19 + 19}$$

$$s_p^2 = \frac{11.59}{38}$$

$$s_p^2 = .305$$

Soccer Study Results, Step 2

Compute the Standard Error of the Difference

$$S_{\overline{X}_1-\overline{X}_2} = \sqrt{\frac{S_P^2}{N_1} + \frac{S_P^2}{N_2}}$$

$$S_{\overline{X}_1-\overline{X}_2} = \frac{S_P}{\sqrt{N_1}} + \frac{S_P}{\sqrt{N_2}}$$

$$S_{\overline{X}_1-\overline{X}_2} = \sqrt{\frac{.305}{20} + \frac{.305}{20}}$$

$$S_{\overline{X}_1-\overline{X}_2} = \sqrt{.0305}$$

$$S_{\overline{X}_1-\overline{X}_2} = .1746$$

Soccer Study Results, Step 3

Compute t.

$$t_{(N_1+N_2-2)} = \frac{\overline{X}_1 - \overline{X}_2}{S_{\overline{X}_1-\overline{X}_2}}$$

$$t_{(38)} = \frac{2.50 - 1.90}{.1746}$$

$$t_{(38)} = \frac{.60}{.1746}$$

$$t_{(38)} = 3.436$$

QUESTION 1: Is this a statistically significant result?

QUESTION 2: What is your conclusion?

Assumptions

1. The observations within each sample are independent.
2. The populations from which the samples are drawn are normally distributed.
3. The populations from which the samples are drawn have equal variances.

Repeated Measures (Within Groups) t-test

Issues

- A significant component of the standard error in the independent groups t-test is random error generated by two separate samples.
- This random error masks any treatment effect.
- One way to control for this is to use the same subjects in both treatment conditions.

Repeated Measures t-test

A study in which participants are measured more than once on the same dependent variable. The same subjects are used in both treatment conditions.

Example

A group of ten freshmen football players were told that they had to bulk up to make the first string squad. They were weighed in September and then put on a strict weight training program with a food supplement designed to increase muscle mass. They were then weighed again in December.

1. What is the research hypothesis?
2. What is H_o?
3. What is the statistical hypothesis?
4. Is this a one-tailed or two-tailed test?

Weight Gain Study

Before	After	Difference
182	177	-5
184	186	2
184	192	8
181	180	-1
187	187	0
180	189	9
179	183	4
171	182	11
184	186	2
180	184	4
	Mean	3.4

Statistical question.

Could these difference scores have happened by chance?

Model

$$t_{(N-1)} = \frac{\overline{D}}{S_{\overline{D}}}$$

$$S_{\overline{D}} = \sqrt{\frac{S_D^2}{N}}$$

$$S_{\overline{D}} = \frac{S_D}{\sqrt{N}}$$

Weight Gain Study, Step 1

Before	After	Difference	$(D - M_D)^2$
182	177	-5	70.56
184	186	2	1.96
184	192	8	21.16
181	180	-1	19.36
187	187	0	11.56
180	189	9	31.36
179	183	4	.36
171	182	11	57.76
184	186	2	1.96
180	184	4	.36
			SS_D = 216.40

Weight Gain Study, Step 2

Compute the Standard Error of D.

$$S_{\overline{D}} = \sqrt{\frac{S_D^2}{N}}$$

$$S_{\overline{D}} = \sqrt{\frac{24.04}{10}}$$

$$S_{\overline{D}} = \sqrt{2.404}$$

$$S_{\overline{D}} = 1.551$$

Weight Gain Study, Step 3

Compute t.

$$t_{(N-1)} = \frac{\overline{D}}{S_{\overline{D}}}$$

$$t_{(9)} = \frac{3.44}{1.551}$$

$$t_{(9)} = 2.23$$

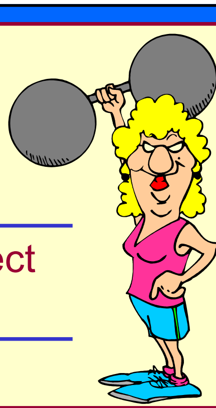
QUESTION 1: What is your statistical conclusion?

QUESTION 2: What conclusion do your reach?

Assumptions

1. Observations within each treatment condition are independent.
2. The population distribution of the difference scores is normal.

Power and Effect Size



Power

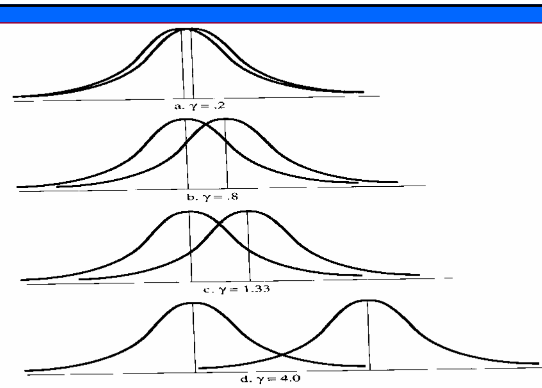
- Can the statistical test detect a treatment difference when the difference exists?
- POWER is the probability that the test will correctly reject a false null hypothesis.
- A weak statistical test will raise the probability of making a Type II error.

Things That Influence Power

1. Alpha level.
2. One vs. two-tailed test.
3. Sample size.

Effect Size

- In experimental research effect size is the magnitude or influence of the independent variable on the dependent variable.
- In correlational research effect size is the strength of the relationship between the variables.



Power and Effect Size

1. A powerful statistical test will detect a weak effect.
2. A weak test will fail to detect a small effect (Type II error).

Statistical vs. Practical Significance

- Statistical significance lets you know whether your results could have happened by chance.
- Practical significance the judgment as to whether the found relationships are meaningful.

Effect Size Assessment

$$d = \frac{\mu_1 - \mu_2}{\sigma}$$

d represents the effect size, the difference between the two means in standard deviation units.

d	Interpretation
.20	Small
.50	Medium
.80	Large

Example: Soccer Study

$$d = \frac{\overline{X_1} - \overline{X_2}}{S_P}$$

$$d = \frac{2.50 - 1.90}{.552}$$

Interpreted in terms of how far apart the two means are in standard deviation units.

$$d = 1.08$$

Omega Squared ($\hat{\omega}^2$)

$$\hat{\omega}^2 = \frac{t_{obt}^2 - 1}{t_{obt}^2 + N - 1}$$

Interpreted in terms of the amount of variability accounted for in the dependent variable when one knows the level of the independent variable.

Soccer Study

$$\hat{\omega}^2 = \frac{3.4316^2 - 1}{3.4316^2 + 20 - 1}$$

$$\hat{\omega}^2 = \frac{10.9716}{30.9716}$$

$$\hat{\omega}^2 = .35$$

eta^2

$$eta^2 = \frac{t^2}{t^2 + df}$$

Interpreted in terms of the amount of variability accounted for in the dependent variable when one knows the level of the independent variable.

Soccer Study

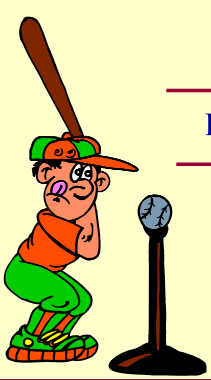
$$eta^2 = \frac{3.436^2}{3.436^2 + 18}$$

$$eta^2 = \frac{11.8061}{29.8061}$$

$$eta^2 = .396$$

Comparison of eta^2 and $\hat{\omega}^2$

	eta^2	$\hat{\omega}^2$
SAT Study	.60	.58
Soccer Study	.40	.35
Weight Gain Study	.35	.35



Practice Problems

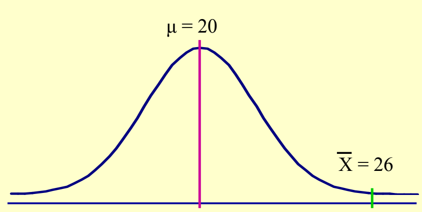
Memory Booster Problem

Sy Napse is planning to sell a memory booster, a concoction of herbs and minerals intended to improve memory performance. Sy intends to test the effectiveness with a sample of 16 people by having them take the mixture daily for six days. At the end of this time they take a standardized memory test.

The sample scores were M=26, s=8. For the population, μ = 20.

- What kind of statistical test should Sy use?
- Is this a one or two-tailed test?
- What is the statistical hypothesis?
- What are the degrees of freedom.
- What is the critical value of t?

Step 1, Draw the Picture



Step 2, Compute the Standard Error of the Mean

$$S_{\overline{X}} = \frac{s}{\sqrt{N}}$$

Step 3. Compute t

$$t_{(N-1)} = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

Step 4. Decide if t is Significant

- Does t exceed the critical value?
  - If 'yes' reject H_o.
  - If 'no' fail to reject H_o.

Step 5. What Conclusions do You Reach?

- If H_o is rejected then your research hypothesis is supported.
- If H_o is not rejected then your research hypothesis is not supported.

Step 6. Compute eta^2

$$eta^2 = \frac{t^2}{t^2 + df}$$

Harassment Seminars

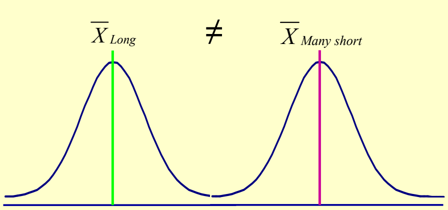
Hanz Zoff, well known sexual harassment counselor wanted to evaluate the impact of one long harassment workshop against many shorter ones.

Hanz used employees at two locations. There were 15 employees in each group. The mean number of complaints for the single session was 72, SS = 100. For the multiple session group the mean was 68, SS = 110.

Did workshop frequency have an impact of harassment?

1. **What statistical test should Hanz use?**
2. **Should this be a one or two-tailed test?**
3. **What are the degrees of freedom?**
4. **What is H_o?**
5. **What is the critical value of t at p<.05?**
6. **What is the critical value at p<.01?**

Step 1. Draw the Picture



Step 2. Compute the Pooled Variance

$$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$$

Step 3. Compute the Standard Error

$$S_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{S_P^2}{N_1} + \frac{S_P^2}{N_2}}$$

Step 4. Compute t

$$t_{(N_1 + N_2 - 2)} = \frac{\overline{X}_1 - \overline{X}_2}{S_{\overline{X}_1 - \overline{X}_2}}$$

Step 5. Decide if t is Significant

Step 6. What Conclusion do You Reach?

Step 7. Compute eta^2

$$eta^2 = \frac{t^2}{t^2 + df}$$

Study Problem

Fitz Mabut was interested in whether students learned better in comfortable chairs versus uncomfortable chairs. Fitz thought that the comfortable chairs would lead to poorer grades. He had five participants study first in comfortable then in uncomfortable chairs.

Fitz evaluated the level of learning under both the comfortable and uncomfortable conditions.

1. **What kind of statistical test should Fitz use?**
2. **Is this a one tailed or two-tailed test?**
3. **What is the statistical hypothesis?**
4. **What are the degrees of freedom?**
5. **What is the critical value of t?**

Comfort Problem, Part 1

Person	Comfortable	Uncomfortable	D	$(D-M_D)^2$
1	16	10		
2	5	3		
3	12	10		
4	9	5		
5	23	15		

Comfort Problem

Compute Standard Error

$$S_{\overline{D}} = \sqrt{\frac{S_D^2}{N}}$$

Compute t

$$t_{(N-1)} = \frac{\overline{D}}{S_{\overline{D}}}$$

THE END