Graduate Statistics

Comparing Two Means: Z-test

What We Will Cover in This Section

- Introduction.
- The Z-test review.
- Other problems.

The Z-Test

Application: The Z-test

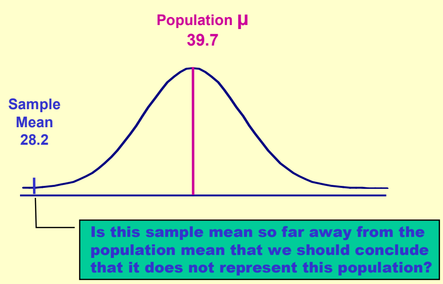
The average age of registered voters in Slippery Gulch is $\mu = 39.7$ years old and the standard deviation, σ, is 10.

The League of Women Voters wanted to encourage younger people to vote so they sponsored a series of educational articles and television commercials on the benefits of voting.

Afterwards, a sample of 12 voters at the latest election was found to have a mean age of 28.2 years.

Did the advertising have an effect on voters or could this result have been a result of random error?

The Statistical Model, Again



How to Think About This

Could a sample with a mean of 28.2 have occurred in a distribution where the mean is 39.7 and the standard deviation is 10?

or

Does the sample with M = 28.2 represent a different population?

What distinguishes this 'different' population would be the commercials.

Decision Issues

- How do you determine far away?
  - What measure do we have to determine how far away a sample mean is from the population mean?
- How do we determine if this mean is rare?
  - What is rare?

The Z-Test Formula

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

How to Compute, Step #1

1. Determine the critical value for a one-tail test where $p < .05$.

Critical Value = -1.64

How to Compute, Step #2

2. Calculate the standard error.

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$$

$$\sigma_{\overline{X}} = \frac{10}{\sqrt{12}}$$

$$\sigma_{\overline{X}} = \frac{10}{3.464}$$

$$\sigma_{\overline{X}} = 2.89$$

How to Compute, Step #3

3. Calculate how far the sample mean is from the population mean in SE units.

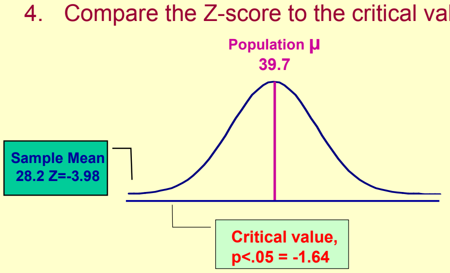
$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

$$Z = \frac{28.2 - 39.7}{2.89}$$

$$Z = -3.98$$

How to Compute, Step #4

4. Compare the Z-score to the critical value.



Properties of the Z-test

- What you can learn.
  Does a sample mean (M) differ significantly from a population mean (μ) or could this difference have occurred by chance.
- Assumptions.
  - Interval or ratio scales.
  - Know μ and σ.
  - Know the sample mean.
  - Know the sample size.

ALPHA Level (α)

- *ALPHA* is the statistical statement of something that is rare.
  - Traditionally, *alpha* is defined as something that would happen 5% of the time or less.
  - This is shown by: $p < .05$.

Critical Values for α

Critical Value	Type of test	
	One tail	Two tailed
.05	1.64	1.96
.01	2.33	2.58

Example #2

Melody Tunne thought that listening to music while taking a statistics test would either be relaxing, increasing performance, or distracting, decreasing performance. She did not know which.

1. Is this a one-tail or two-tail test?
2. What alpha level should Melody set?

Melody's Data

- The mean for the population of students who have taken the statistics test is $\mu = 50$.
- The standard deviation for all students is $\sigma = 12$.
- Melody got a sample of 49 students who listened to music while taking the test.
  - Their mean was 54.63
  - Their standard deviation was 7.

The END?

T-tests