Graduate Statistics

Analysis of Variance

ANOVA

What We Will Cover in This Section

- Introduction.
- Overview.
- Simple ANOVA.
- Repeated Measures ANOVA.
- Factorial ANOVA

Situation

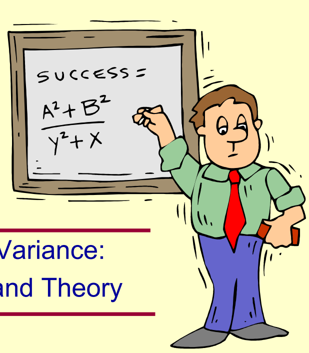
The management of Sal T. Dogg's restaurant wanted to see if the saltiness of appetizers would influence the number of drinks people purchased. Three sections of the club are targeted to receive appetizers that have either low, medium, or high saltiness. The dependent variable is the number of drinks ordered.

1. What is the research hypothesis?
2. What is H_o?
3. What is the statistical hypothesis?

Appetizer saltiness and number or drinks ordered.		
Group 1 Low Salt	Group 2 Medium Salt	Group 3 High Salt
2	3	3
3	4	1
1	4	2
1	5	2
2	6	3
2	4	1
1	3	1
2	2	2
2	4	1
4	4	2
M = 2.00	M = 3.90	M = 1.80

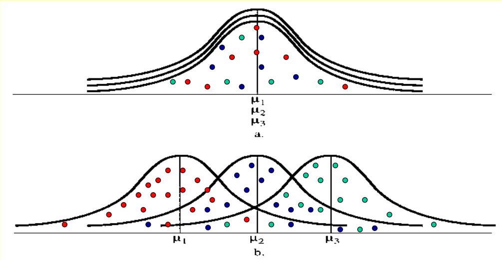
Issue

How to determine if one mean is significantly different from the other means while minimizing the probability of committing a Type I error.

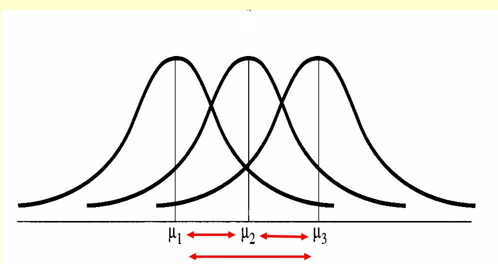


Analysis of Variance: Background and Theory

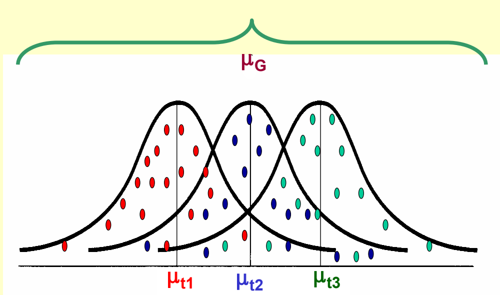
Logic

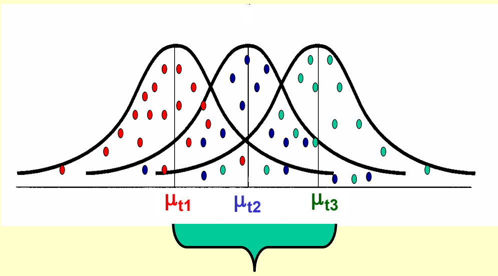


Treatment Effects



Total Variability



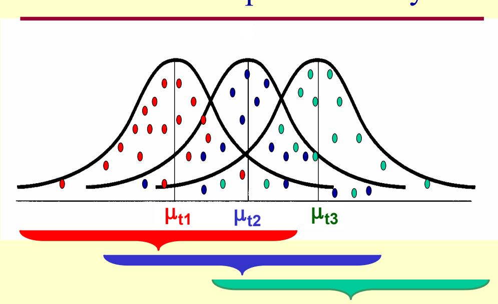


Between Groups Variability

Sources of Between Group Variability

- TREATMENT effect
- Random error from...
  - Subjects.
  - Measurement.
  - Random.

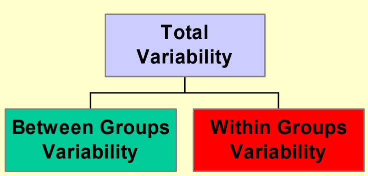
Within Groups Variability



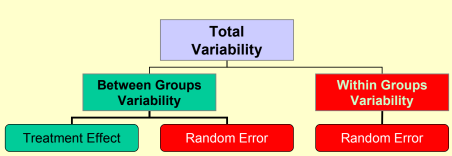
Sources of Within Group Variability

- Random Error from…
  - Subjects
  - Measurement.
  - Random.

Partitioning the Variance



Partitioning the Variance



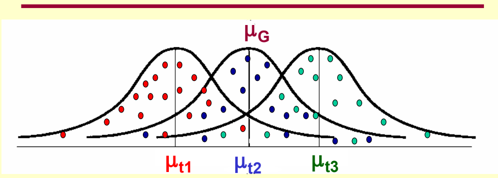
Where We Are Going

$$\frac{(\text{Treatment}) + (\text{Random Error})}{\text{Random Error}}$$

F-test Compared to t-test

$$t_{(df)} = \frac{X_1 - X_2}{\sigma_{\bar{X}}} \qquad F_{(k-1,N-k)} = \frac{MS_B}{MS_W}$$

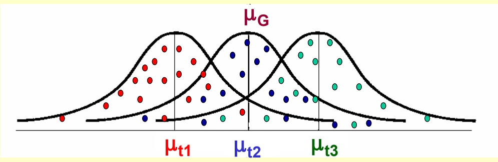
Partitioning the Variability



$$\frac{\sum(X-\mu_G)^2}{N-1} = \frac{\sum(\mu_T-\mu_G)^2}{k-1} + \Sigma\left(\frac{\sum(X-\mu_T)^2}{N-k}\right)$$

Total = Between + Within

ANOVA Model



Total Effect	=	Treatment Effect	+	Random Error
Total Variability	=	Between Groups Variability	+	Within Groups Variability

Partitioning the Variance

Total Variability = Between Groups Variability + Within Groups Variability

$$\frac{SS_{Total}}{df_{Total}} = \frac{SS_{Between\,Groups}}{df_{Between}} + \frac{SS_{Within\,Groups}}{df_{Within}}$$

(N - 1) (k – 1) (N – k)

F-test Model

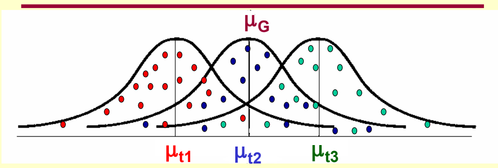
Between Groups Variability

- TREATMENT EFFECT
- Subject Error
- Measurement Error
- Random Error

Within Groups Variability

- Subject Error
- Measurement Error
- Random Error

The F ratio

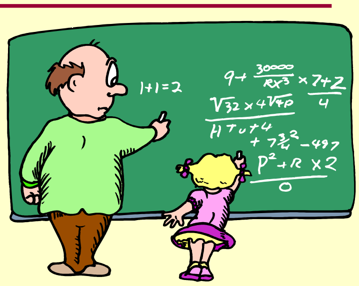


$$F_{(k-1,N-k)} = \frac{MS_B}{MS_W} = \frac{\text{Between Groups}}{\text{Within Groups}}$$

The Critical Value of F

- See page 695 in old text book, 693 in new text book.
- Notice
  - Need *df* $_{between}$ (numerator) for columns.
  - Need *df* $_{within}$ (denominator) for rows.
  - As *df* increases the critical values get smaller.

The Problem Returns



The Situation (in case you forgot)

The management of Sal T. Dogg's restaurant wanted to see if the saltiness of appetizers would influence the number of drinks people purchased. Three sections of the club are targeted to receive appetizers that have either low, medium, or high saltiness. The dependent variable is the number of drinks ordered.

Hypotheses

Research Hypothesis.

Saltiness of the appetizers will influence the number of drinks that people buy.

Null Hypothesis.

Saltiness will not influence the number of drinks that people buy.

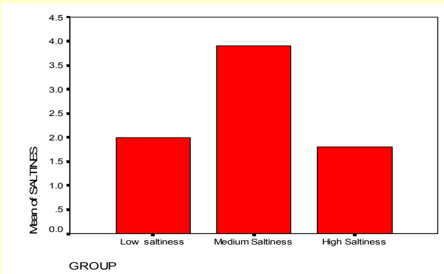
Statistical Hypothesis.

$\mu_1 \neq \mu_2 \neq \mu_3$

Appetizer saltiness and number or drinks ordered.

Group 1 Low Salt	Group 2 Medium Salt	Group 3 High Salt
2	3	3
3	4	1
1	4	2
1	5	2
2	6	3
2	4	1
1	3	1
2	2	2
2	4	1
4	4	2
M = 2.00	M = 3.90	M = 1.80

Graph of Saltiness Ratings



ANOVA Summary Table

Source	SS	df	MS	$F_{(crit=3.35)}$
Between Groups	26.87	2	13.435	14.77
Within Groups	24.50	27	.91	
Total	51.37	29		

How to Express F

$F_{(2,27)} = 14.77, \; p<.05$

- Degrees of freedom (between, within)
- Calculated F
- Alpha

Post Hoc Tests

When F is significant, how do you determine which of the means differs from the others?

Tukey Honestly Significant Difference Test (HSD)

$$HSD = q_{(\alpha, df_{within}, k)} \sqrt{\frac{MS_{within}}{n}}$$

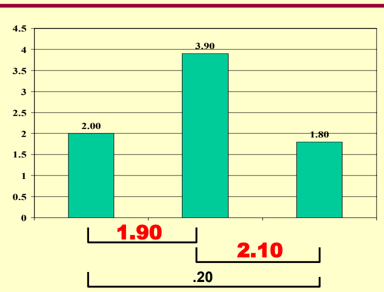
q = Value from table
α = desired significance level
df_{within} = within groups df
k = Number of groups.

$$HSD = q_{(.05,27,3)} \sqrt{\frac{.91}{10}}$$

$$HSD = 3.53 x \sqrt{.091}$$

$$HSD = 1.06$$

How To Use Tukey (*HSD = 1.06*)



Scheffé Test

- Compute a value called C for each pair of means.
- C corrects for multiple pairwise comparisons.
- Need to compute C only once if the sample sizes are equal for all groups.
- To make a decision you compare the computed C_{obt} to C_{crit}.

Computing C_{crit}

$$C_{crit} = \sqrt{(k-1)(F_{crit})}$$

$$C_{crit} = \sqrt{(2)(3.37)}$$

$$C_{crit} = 2.596$$

If the computed value of C exceeds the critical value, then the two means are significantly different based on the alpha level of F_{crit}.

Computing C_{obt}

$$C_{obt} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{MS_W\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$C_{obt} = \frac{2.00 - 3.90}{\sqrt{.91\left(\frac{1}{10} + \frac{1}{10}\right)}}$$

$$C_{obt} = \frac{-1.90}{\sqrt{.182}}$$

$$C_{obt} = -4.45$$

Advantages of Scheffé

- Evaluates each pair of means at a time.
- Corrects for differing sample sizes.
- More conservative than Tukey.

Effect Size: Eta Squared (η^2)

$$\eta^2 = \frac{SS_{Treatment}}{SS_{Total}}$$

$$\eta^2 = \frac{26.87}{51.37}$$

$$\eta^2 = .523$$

Effect Size: Omega Squared $\hat{\omega}^2$

$$\hat{\omega}^2 = \frac{SS_B - (K-1)MS_W}{SS_T + MS_W}$$

$$\hat{\omega}^2 = \frac{26.87 - (3-1)x.91}{51.37 + .91}$$

$$\hat{\omega}^2 = \frac{25.05}{52.28}$$

$$\hat{\omega}^2 = .479$$

Assumptions

1. The observations within each sample are independent.
2. The population from which the samples are selected is normally distributed.
3. The population from which the samples are selected have equal variances (homogeneity of variance)

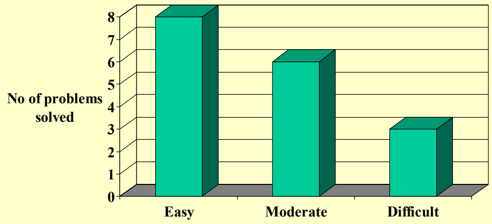
Another ANOVA Example

Sal O. Gysm felt that the perceived difficulty of logic problems would influence performance on these problems. Sal developed a set of problems and gave them to three groups. One group was told that the problems was easy, another was told that they were moderately difficult, and the third was told that they were difficult. The dependent variable was the number of problems solved.

ANOVA: Example 2

Easy	Moderate	Difficult
9	4	1
12	6	3
4	3	4
8	2	5
7	10	2
M = 8.0	M = 6.0	M = 3.0

Logic Problem Results



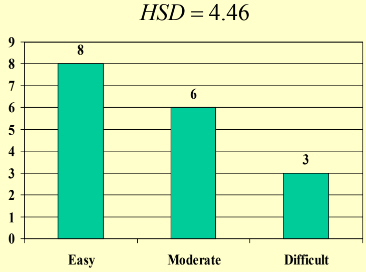
ANOVA: Summary Table

Source	SS	df	MS	F
Between	63.33	2	31.67	4.52*
Within	84.00	12	7.00	
Total	147.33	14		

* $p < .05$

Post hoc Analysis: Tukey HSD

$HSD = 4.46$



Effect Size 1: Eta²: (η^2)

$$\eta^2 = \frac{SS_{between}}{SS_{total}}$$

$$\eta^2 = \frac{63.33}{147.33}$$

$$\eta^2 = .428$$

Effect Size 2: Omega²

$$\hat{\omega}^2 = \frac{SS_B - (K-1)MS_W}{SS_T + MS_W}$$

$$\hat{\omega}^2 = \frac{63.33 - (2)7.00}{147.33 + 7.00}$$

$$\hat{\omega}^2 = \frac{40.33}{157.33}$$

$$\hat{\omega}^2 = .255$$

Another Practice Problem

Tess Tosterone is studying aggression among adolescent girls. She believes that there is a relationship between the level of interaction a girl has with her mother and the girl's level of aggression. She has identified fifteen girls who fall into one of three maternal interaction levels (low, medium, and high) and has measured their aggression scores.

The scores are shown on the next slide.

Data Summary Table

Low interaction	Moderate Interaction	High Interaction
6	6	0
5	8	4
9	5	0
4	4	1
6	2	0
M = 6.00	M = 5.00	M = 1.00

ANOVA Summary Table

	Sum of Squares	df	Mean Square	F
Between	70.00	2	35.00	9.13
Within	46.00	12	3.833	
Total	116.00	14		

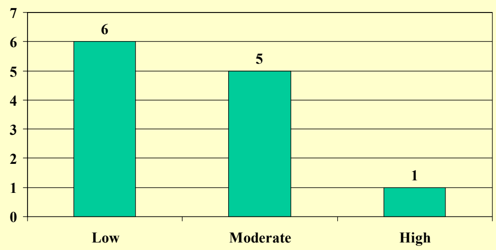
Tukey HSD

$$HSD = q_{(\alpha, df_{within}, k)} \sqrt{\frac{MS_{within}}{n}}$$

$$3.77\sqrt{\frac{3.833}{5}}$$

HSD = 3.30

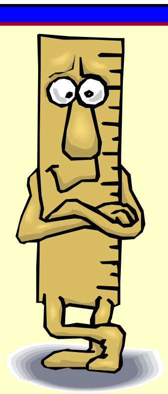
Which Means are Different?



Eta^2

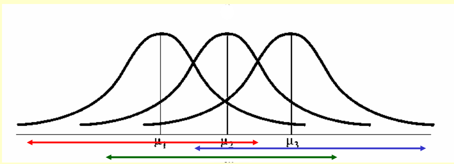
$$\eta^2 = \frac{SS_{between}}{SS_{total}} \qquad \frac{70.00}{116.00}$$

.60



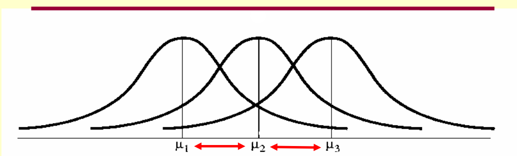
Repeated Measures ANOVA

Sources of **Within Group** Variability



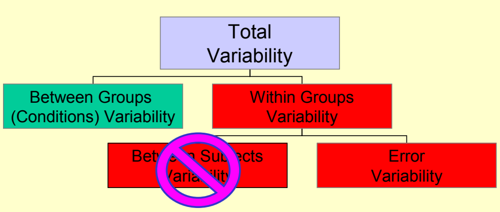
1. Measurement error.
2. Individual differences among the subjects.
3. Random error.

Sources of **Between Group** Variability



1. TREATMENT EFFECT.
2. Individual differences among the subjects.
3. Measurement Error.
4. Random error.

Partitioning the Variance



Partitioning the Variance

Total Variability = Between Groups Variability + Between Subjects Variability + Error Variability

$$\frac{SS_{Total}}{df_{Total}} = \frac{SS_{Between}}{df_{Between}} + \frac{SS_{Between\,Ss}}{df_{Between\,Ss}} + \frac{SS_{Error}}{df_{Error}}$$

(N - 1) (N - k) (n – 1) (N – k) - (n - 1)

The F-test

$$F_{(k-1)(n-1)} = \frac{MS_{Treatment}}{MS_{Error}}$$

Example: Relaxation Therapy

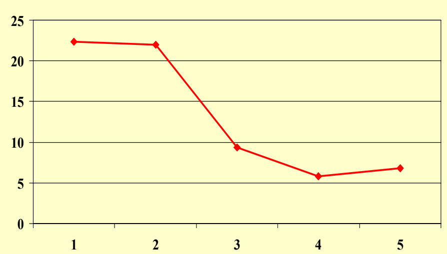
Nine migraine sufferers were asked to document the strength of their headaches. There was a two-week baseline period followed by three weeks of relaxation therapy.

The therapists wanted to determine if the therapy was effective.

1. What is the research hypothesis?
2. What is H_o?
3. What is the statistical hypothesis?

	Baseline week		Treatment Week			Subject
Subject	1	2	3	4	5	Total
1	21	22	8	6	6	63
2	20	19	10	4	4	47
3	17	15	5	4	5	46
4	25	30	13	12	17	97
5	30	27	13	8	6	84
6	19	27	8	7	4	65
7	26	16	5	2	5	54
8	17	18	8	1	5	49
9	26	24	14	8	9	81
Mean	22.33	22.00	9.33	5.78	6.78	13.24

Mean Headache Strength by Week



Summary Table

Source	*df*	SS	MS	F
Between				
Weeks	4	2449.20	612.30	85.04
Within				
Between Subjects	8	486.71		Error
Error	32	230.40	7.20	
Total	44	3166.31		

Post hoc Tests

- Tukey's HSD
  - Replace MS_{within} with Ms_{error}.
  - Replace df_{within} with df_{error}.
- Scheffé
  - Replace MS_{within} with Ms_{error}.
  - Replace df_{within} with df_{error}.

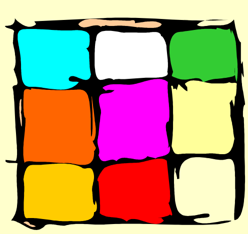
Effect Size

$$\eta^2 = \frac{SS_{between}}{SS_{total}}$$

$$\eta^2 = \frac{2449.2}{3166.31}$$

$$\eta^2 = .774$$

Factorial ANOVA



Definition

Experimental design in which there are two or more independent variables and one dependent variable.

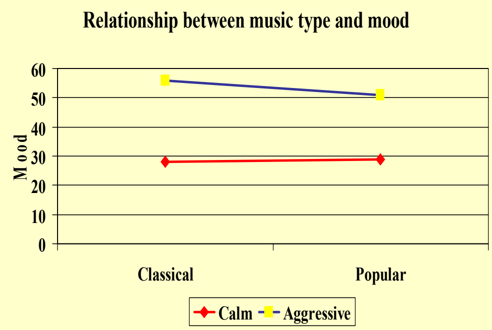
Problem #1 Effects of Music on Mood

Clarissa Thompson was researching the influence of music on mood. She hypothesized that tone of the music (aggressive vs. calm) would influence a person's mood but that the type of music (classical vs. popular) would not affect mood.

She randomly divided 60 volunteers into one of four groups: classical-aggressive, classical-calm, popular-aggressive, or popular-calm. Then she played a six-minute musical selection for the person then had them rate their mood.

Music Study Descriptive Statistics

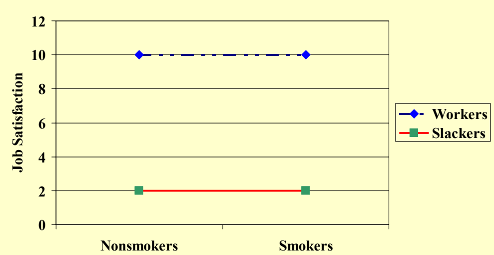
	Music Type		
	Aggressive	Calm	
Classical	56.00	28.27	42.14
Popular	51.29	29.73	40.51
	53.64	29.00	41.32



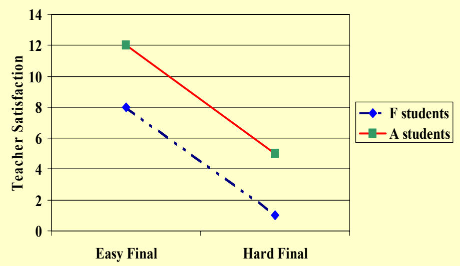
Main Effect

The independent influence that one independent variable alone has on the dependent variable.

Factorial ANOVA: One Main Effect



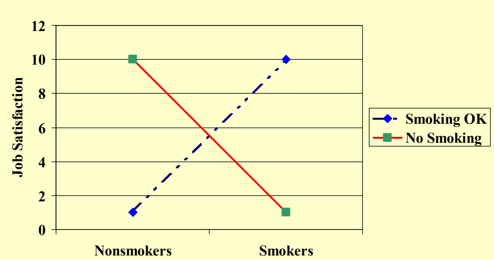
Factorial ANOVA: Main Effects



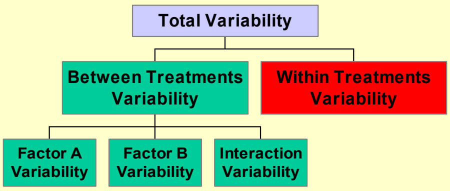
Interaction

The combined effects of two or more independent variables on the dependent variable.

Factorial Graphs: Interaction



Partitioning Sources of Variability



The Problem 2: Chocolate Chip Study

The Home for Retired College Professors (HRCP) wants to do a fund raiser using the expertise of its residents as business consultants. After a trial, the clients complained that the advice was too impractical and academic. The director, Gerry Atric, wants to see if feeding these oldsters with chocolate chips would increase the practicality of their recommendations.

Atric felt that teaching experience would also have an impact on the treatment effect, so she divided the group into those who taught more than 20 years and those who taught less than 20 years.

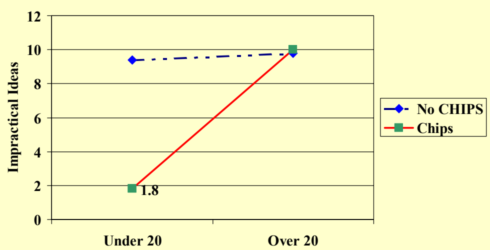
Model

Chocolate Chips

Experience	No	Yes
Under 20 years	n=5	n=5
Over 20 years	n=5	n=5

	No Chips	Chips	Mean
Under 20	9, 10, 8, 9, 11 M= 9.4	2, 3, 2, 1, 1 M= 1.8	5.6
Over 20	10, 11, 9, 9, 10 M= 9.8	9, 12, 10, 9, 10 M= 10.0	9.9
Mean	9.6	5.9	7.75

Chocolate Chip Study



2x2 Factorial ANOVA

Chocolate study summary table

Source	SS	*df*	MS	F
Between Group	236.95	3		
Experience	92.45	1	92.45	88.05*
Chocolate Chips	68.45	1	68.45	65.19*
AXB	76.05	1	76.05	72.43*
Within Group	16.80	16	1.05	
Total	253.75	19		

* $p < .01$

Effect Size

	$\hat{\omega}^2$	η^2
Experience	.333	.338
Chips	.245	.250
Experience x Chips	.273	.278

Factorial ANOVA: Notation

Number of independent variables

4 x 3 x 2 factorial ANOVA

Levels of each independent variable.

Factorial ANOVA Assumptions

1. The observations within each treatment condition are independent.
2. The population distribution is relatively normal.
3. The variances within each treatment condition are equal.


THE END
8/21/2007
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