Psy 331
Inferential Statistics

Inferential Statistics: An Introduction

What We Will Cover in This Section

- Introduction.
- Probability and the Normal Curve.
- Probability and Sampling Means.
- The Z-test.

Research Question

Dr. Hezzy Tater wanted to know if having students set personal goals would help them overcome their natural tendency to put off writing papers. Dr. Tater had one group of students set clear dates for starting and completing their paper for her class. The other group did not set goals. At the end of the semester Tater asked the students in each group when they completed their papers.

Tater predicted that if her goal setting worked, this group would have completed their papers earlier.

Potential Outcome #1

The goal setting does not work.

The goal setting group and regular group represent the same population.

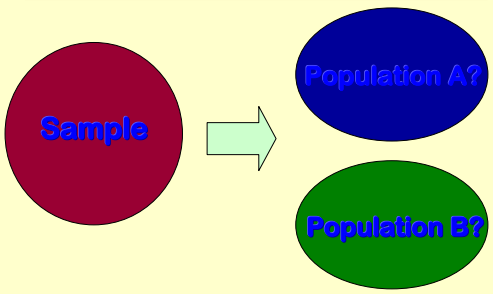
Potential Outcome #2

The goal setting does work.

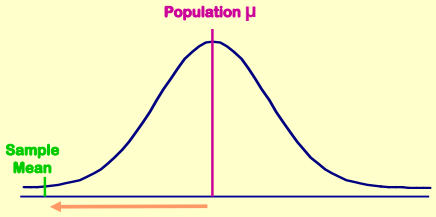
The regular group represents one population.

The goal-setting group represents another population.

General Model



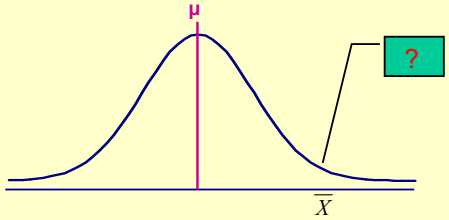
The Statistical Model



Probability and Sampling Means

Key Concept

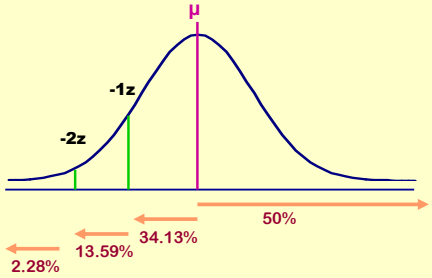
What is the probability that a sample mean came from a particular population?



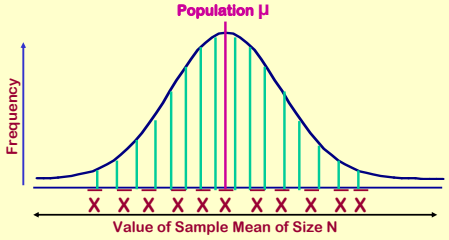
Issue

To what degree does a sample mean represent the population to which we want to make inferences?

Slicing the Normal Curve, Again



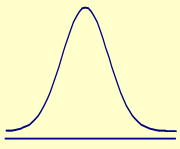
Distribution of Sample Means



Incredibly Cool Demonstration

Distribution of Sample Means

Small Sample



Large Sample

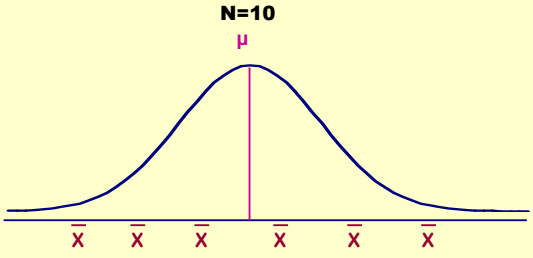
Sampling Error

The degree to which a sample statistic deviates from its corresponding population parameter.

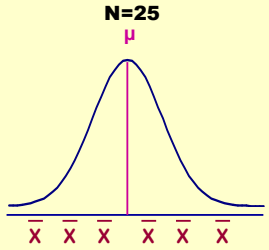
Sampling Distribution

The distribution of statistics of a fixed size (N) taken from a population.

When the Sample Size is Small



When the Sample Size is Large



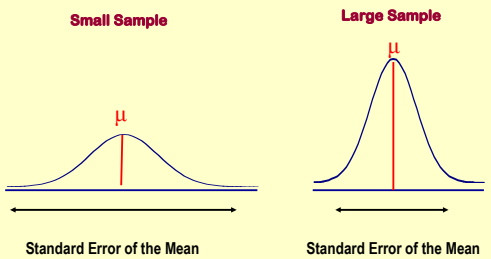
Central Limit Theorem

- When a large number of sample means of size N are selected from a population they will be normally distributed.
- The sampling distribution of the means gives a normal distribution.
- The mean of a sampling distribution of means equals the population μ.

KEY POINT

- As the **sample size gets larger**, the **standard deviation** of the **sample means** gets **smaller**.
- The standard deviation of the sample means is called the **STANDARD ERROR OF THE MEAN**.

Standard Error of the Mean, $\sigma_{\bar{X}}$



Standard Error of the Mean

The standard deviation of the distribution of sample scores around the population mean.

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$$

Example

What is the standard error of the mean for the following?

$\sigma_X = 4$; N = 25

$\sigma_{\bar{X}} = \frac{4}{\sqrt{25}}$

$\sigma_{\bar{X}} = \frac{4}{5}$

$\sigma_{\bar{X}} = .80$

$\sigma_X = 4$; N = 16

More Practice

Given

σ = 24

N = 64

What is $\sigma_{x?}$

Given

σ = 24

N = 36

What is $\sigma_{x?}$

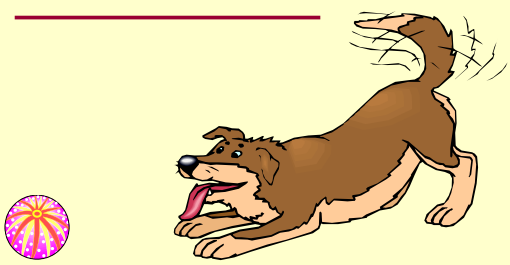
Key Learning Points #1

1. The normal distribution can be used to describe the distribution of many naturally occurring variables.
2. The CLT tells us that the distribution of sample means approximates the normal distribution.
3. The standard deviation divides the normal distribution into meaningful units.
4. The standard deviation of sample means is called the **Standard Error of the Mean**.

Key Learning Points #2

5. The size of the Standard Error of the Mean depends on the sample size.
6. We can describe the probability of randomly selecting any score from the normal distribution. All we need to know is how far that score is from the mean in standard deviation units.
7. We can describe the probability of randomly selecting a mean from a sample of means. All we need to know is the standard error of that mean.

A Tale of Tails



Research Predictions

1. One can predict that one group is bigger than another. This is called a one-tailed (directional) prediction.
2. You can predict that two groups differ but you don't know which will be bigger. This is called a two-tailed (non-directional) prediction.

One-tail Prediction

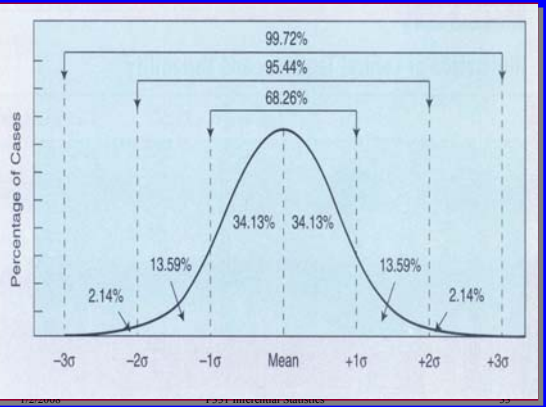
- A research hypothesis where one group is predicted to be larger than another.
- Example.
  - Continuous reinforcement will lead to faster learning than intermittent reinforcement.
  - Babies who are held by their parents will have higher self esteem than babies who are not held by their parents.
  - Students who cram will have lower grades than students who don't cram.

Two-tail Prediction

- A research hypothesis where one group is predicted to be different than another but the researcher does not know if they will be higher or lower.
- Example.
  - Taking a nap before a meal will either increase or decrease your appetite.
  - Listening to music while taking a test will either help or hurt your grade.
  - Students who wear a bow tie to class will either impress or irritate their handsome instructor.

Another Phenomenonally Cool Demonstration

What is Rare?



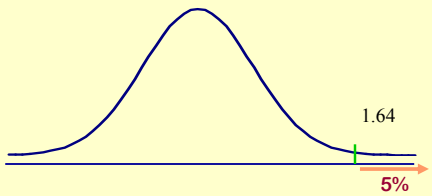
What Is Rare?

Something that would happen less than 5% of the time is 'rare' from the statistician's point of view.

Key Probabilities (Critical Values)

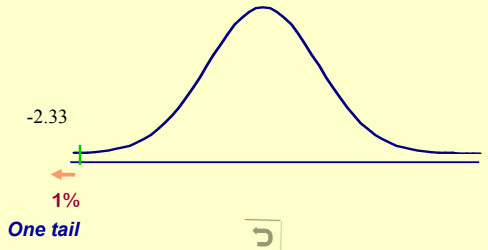
Above what z-score do 5% of the cases fall?	1.64
Below what z-score does 1% of the cases fall?	-2.33
Between which two z-scores do 95% of the cases fall?	±1.96
Between which two z-scores do 99% of the cases fall?	± 2.58

Above what z-score do 5% of the cases fall?



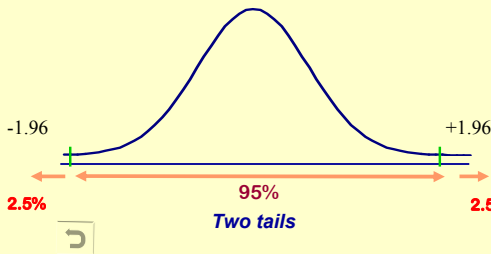
One tail

Below what z-score does 1% of the cases fall?



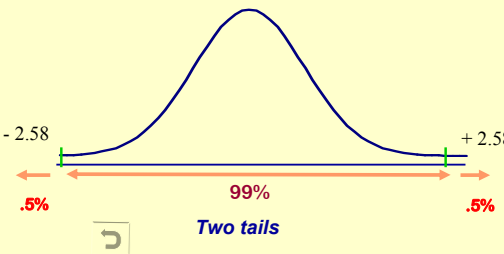
One tail

Between what two z-scores do 95% of the cases fall?



Two tails

Between what two z-scores do 99% of the cases fall?



Two tails

Critical Value

A numerical index that allows us to decide whether a statistical results is rare.

Values for *RARE*

In statistics we use the following values to determine if something is rare.

- **5% of the time or less is moderately rare.**
- **1% of the time or less is very rare.**

How it all works...

The Z-Test

Application: The Z-test

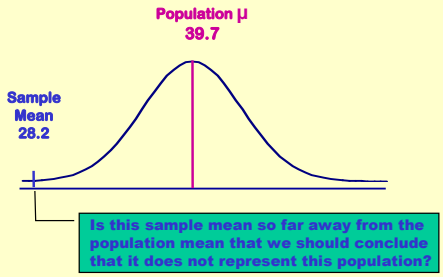
The average age of registered voters in Slippery Gulch is $\mu = 39.7$ years old and the standard deviation, σ, is 10.

The League of Women Voters wanted to encourage younger people to vote so they sponsored a series of educational articles and television commercials on the benefits of voting.

Afterwards, a sample of 12 voters at the latest election was found to have a mean age of 28.2 years.

Did the advertising have an effect on voters or could this result have been a result of random error?

The Statistical Model, Again



How to Think About This

Could a sample with a mean of 28.2 have occurred in a distribution where the mean is 39.7 and the standard deviation is 10?

or

Does the sample with M = 28.2 represent a different population?

What distinguishes this 'different' population would be the commercials.

Decision Issues

- How do you determine far away?
  - What measure do we have to determine how far away a sample mean is from the population mean?
- How do we determine if this mean is rare?
  - What is rare?

The Z-Test Formula

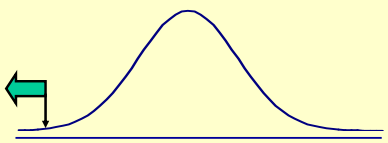
$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

What do we call this thing?

How to Compute, Step #1

1. Determine the critical value for a one-tail test where p < .05. This is the **RARE** value.

Critical Value = -1.64



How to Compute, Step #2

2. Calculate the standard error.

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$$

$$\sigma_{\bar{X}} = \frac{10}{\sqrt{12}}$$

$$\sigma_{\bar{X}} = \frac{10}{3.464}$$

$$\sigma_{\bar{X}} = 2.89$$

How to Compute, Step #3

3. Calculate how far the sample mean is from the population mean in SE units.

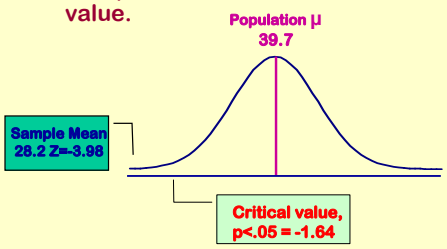
$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

$$Z = \frac{28.2 - 39.7}{2.89}$$

$$Z = -3.98$$

How to Compute, Step #4

4. Compare the Z-score to the critical value.



Properties of the Z-test

- What you can learn.
  - Does a sample mean (M) differ significantly from a population mean (μ) or could this difference have occurred by chance.
- Assumptions.
  - Interval or ratio scales.
  - Know μ and σ.
  - Know the sample mean.
  - Know the sample size.

ALPHA Level (α)

- ***ALPHA*** **is the statistical statement of something that is rare.**
  - **Traditionally, *alpha* is defined as something that would happen 5% of the time or less.**
  - **This is shown by: p < .05.**

Critical Values for α

Alpha	Critical Values	
	One tail	Two tailed
.05	1.64	1.96
.01	2.33	2.58

Assignment

Homework #7

The END?

Hypothesis Testing