Psychological Statistics

z-Scores

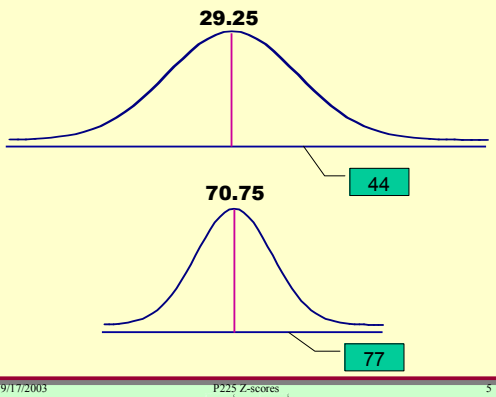
What We Will Cover in This Section

- What a z-score is.
- Computation.
- Properties.
- Assumptions.
- Uses

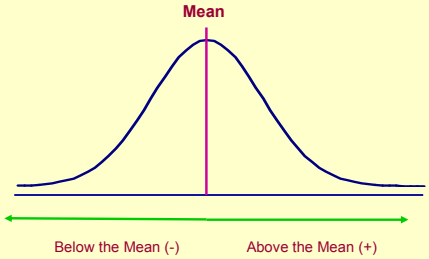
Question

On which of the following tests did Pat do best compared to the other students?

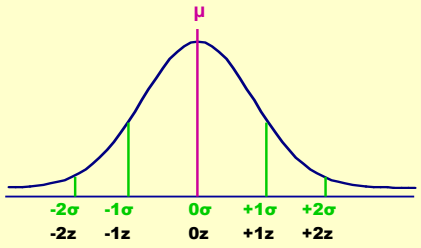
Name	Flea-flicking test	Gnome-naming test
Kim	33	72
Jan	18	66
Fran	22	68
Pat	44	77
Mean	29.25	70.75
SD	11.70	4.86



Concept



Units of Measurement



Equations

Defining Equation

$$z = \frac{x - \mu}{\sigma_X}$$

Working Equation

$$z = \frac{x - \overline{X}}{S_X}$$

Examples

Mean = 50 SD = 10

What is the z score for a raw score of 65?

Z = (65 –50)/10

Z = 15/10

Z = 1.5

What is the z score for a raw score of 45?

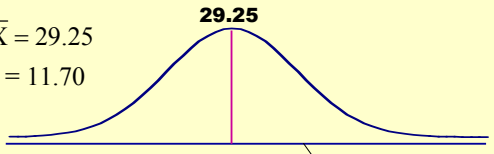
Z = (45 – 50)/10

Z = -5/10

Z = -.5

$\overline{X} = 29.25$

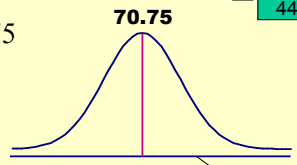
$s = 11.70$



44 Z=1.26

$\overline{X} = 70.75$

$s = 4.86$



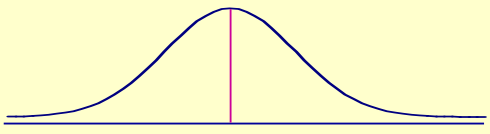
77 Z=1.29

Z-score

The purpose of the z-score is to describe the location of every score in a distribution relative to the mean.

Major Hint

Draw the picture!



z-score to Raw Score

$$\text{Raw Score} = (\text{z-score} \times S_X) + \overline{X}$$

Examples

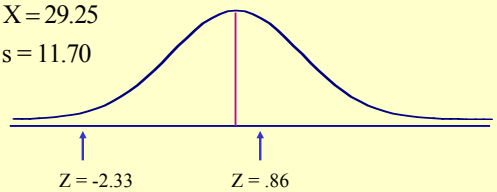
Mean = 35 SD = 4

What is the raw score for a z-score of 1.5?	What is the raw score for a z-score of -.62?
Raw = (1.5 x 4)+ 35	Raw = (-.62 x 4) + 35
Raw = 6 + 35	Raw = -2.48 + 35
Raw = 41	Raw = 32.52

Compute the Raw Scores

$\overline{X} = 29.25$

$s = 11.70$



Z Score to New Raw Score (Standardized Scores)

$$X_{New} = (Z \; x \; SD_{New}) + \overline{X}_{New}$$

Example

Mean = 100 SD = 16

What is the IQ score for a z-score of 1.5?

IQ = (1.5 x 16) + 100

IQ = 24 + 100

IQ = 124

What is the IQ score for a z-score of -1?

IQ = (-1 x 16) + 100

IQ = -16 + 100

IQ = 84

Properties of z-Score

1. The mean of the z distribution is 0.
2. The standard deviation of the z distribution is 1.00.
3. The z-score always indicates how far a score is from the mean. The units of measurement are standard deviation units.
4. The shape of the z-distribution will be the same as the parent distribution.

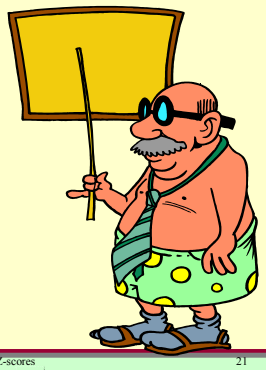
Assumptions

1. The distribution is normal.
2. The units of measurement are interval or ratio scales.

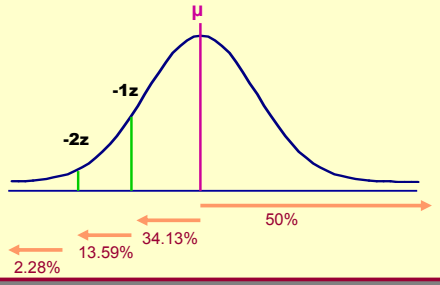
Uses of the z-score

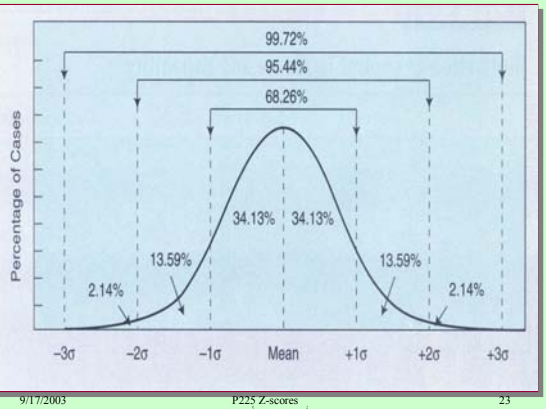
- Comparing different people on the same test.
- Comparing same person across different measures.
- Comparing different people across different tests.

Using z-scores

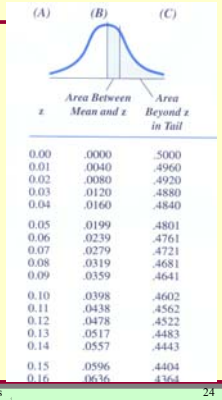


Slicing the Normal Curve





Heiman, Table 1, p. 478



z	Area Between Mean and z	Area Beyond z in Tail
0.00	.0000	.5000
0.01	.0040	.4960
0.02	.0080	.4920
0.03	.0120	.4880
0.04	.0160	.4840
0.05	.0199	.4801
0.06	.0239	.4761
0.07	.0279	.4721
0.08	.0319	.4681
0.09	.0359	.4641
0.10	.0398	.4602
0.11	.0438	.4562
0.12	.0478	.4522
0.13	.0517	.4483
0.14	.0557	.4443
0.15	.0596	.4404
0.16	.0636	.4364

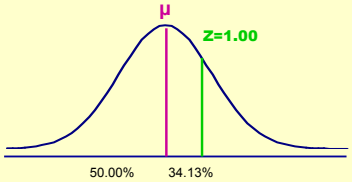
(A) z	(B) Area Between Mean and z	(C) Area Beyond z in Tail	(A) z	(B) Area Between Mean and z	(C) Area Beyond z in Tail	(A) z	(B) Area Between Mean and z	(C) Area Beyond z in Tail
0.00	.0000	.5000	0.30	.1179	.3821	0.60	.2257	.2743
0.01	.0040	.4960	0.31	.1217	.3783	0.61	.2291	.2709
0.02	.0080	.4920	0.32	.1255	.3745	0.62	.2324	.2676
0.03	.0120	.4880	0.33	.1293	.3707	0.63	.2357	.2643
0.04	.0160	.4840	0.34	.1331	.3669	0.64	.2389	.2611
0.05	.0199	.4801	0.35	.1368	.3632	0.65	.2422	.2578
0.06	.0239	.4761	0.36	.1406	.3594	0.66	.2454	.2546
0.07	.0279	.4721	0.37	.1443	.3557	0.67	.2486	.2514
0.08	.0319	.4681	0.38	.1480	.3520	0.68	.2517	.2483
0.09	.0359	.4641	0.39	.1517	.3483	0.69	.2549	.2451
0.10	.0398	.4602	0.40	.1554	.3446	0.70	.2580	.2420
0.11	.0438	.4562	0.41	.1591	.3409	0.71	.2611	.2389
0.12	.0478	.4522	0.42	.1628	.3372	0.72	.2642	.2358
0.13	.0517	.4483	0.43	.1664	.3336	0.73	.2673	.2327
0.14	.0557	.4443	0.44	.1700	.3300	0.74	.2704	.2296
0.15	.0596	.4404	0.45	.1736	.3264	0.75	.2734	.2266
0.16	.0636	.4364	0.46	.1772	.3228	0.76	.2764	.2236
0.17	.0675	.4325	0.47	.1808	.3192	0.77	.2794	.2206
0.18	.0714	.4286	0.48	.1844	.3156	0.78	.2823	.2177
0.19	.0753	.4247	0.49	.1879	.3121	0.79	.2852	.2148
0.20	.0793	.4207	0.50	.1915	.3085	0.80	.2881	.2119
0.21	.0832	.4168	0.51	.1950	.3050	0.81	.2910	.2090
0.22	.0871	.4129	0.52	.1985	.3015	0.82	.2939	.2061
0.23	.0910	.4090	0.53	.2019	.2981	0.83	.2967	.2033
0.24	.0948	.4052	0.54	.2054	.2946	0.84	.2995	.2005
0.25	.0987	.4013	0.55	.2088	.2912	0.85	.3023	.1977
0.26	.1026	.3974	0.56	.2123	.2877	0.86	.3051	.1949
0.27	.1064	.3936	0.57	.2157	.2843	0.87	.3078	.1922
0.28	.1103	.3897	0.58	.2190	.2810	0.88	.3106	.1894
0.29	.1141	.3859	0.59	.2224	.2776	0.89	.3133	.1867

Questions

1. What percent of the scores fall above a z-score of 1?
2. What percent of scores fall between the mean and a z-score of .5?
3. What percent of scores fall below a z-score of -1.8?
6. What percent of scores fall between the mean an z = -.6?
7. What percent of scores fall above z = 1.58?
8. What percent of scores fall between z=1.64 and z= -1.64?

More Questions

1. What percent of scores fall below a z-score of 1.00?



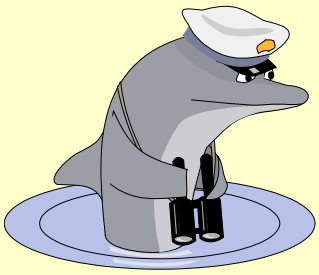
More Questions

1. What percent of scores fall below a z-score of .55?
2. What percentile is represented by a z-score of –1.77?
3. What percent of scores fall between z = 1.00 and z = -.55?

Even More Questions!

- Dr. Digit gave a mathematics test with a mean of 77 and standard deviation of 6.
  - Kim had a score of 80. What was Kim's percentile?
  - Pat had a score of 70. What was Pat's percentile?
  - What score falls at the 95th percentile?

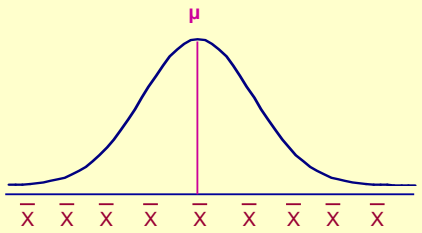
Looking Ahead



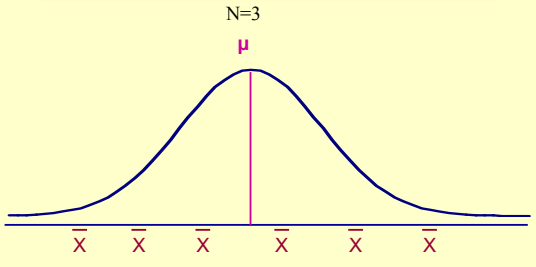
Sampling Means: Introduction

- $\bar{X}$ represents a sample mean.
- How do we estimate a population the mean, μ?

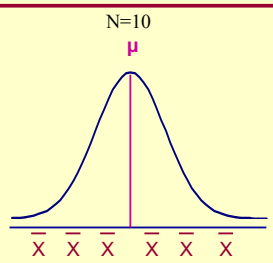
Sampling Distribution of the Mean



When the Sample Size is Small



When the Sample Size is Large



Central Limit Theorem

- The sampling distribution of the means gives a normal distribution.
- The mean of a sampling distribution of means equals the population μ.

Standard Error of the Mean

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{N}}$$

What is the Standard Error of the Mean?

- The Standard Error of the Mean is the standard deviation of sample means.
- It is different for each sample size.
- As the sample size gets bigger the standard error gets smaller.

Example

What is the standard error of the mean for the following?

$\sigma_X = 4; N = 25$ $\sigma_X = 4; N = 16$

$$\sigma_{\bar{X}} = \frac{4}{\sqrt{25}}$$

$$\sigma_{\bar{X}} = \frac{4}{5}$$

$$\sigma_{\bar{X}} = .80$$

Key Learning Points, Part 1

1. Z-scores show how far a score is from the mean.
2. Z-scores assume that the distribution is normal.
3. Z-scores can be transformed to standard scores.

Key Learning Points, Part 2

4. Sample means are normally distributed.
5. The standard deviation of sample means is called the standard error of the mean.
6. The standard error of the mean depends on sample size.