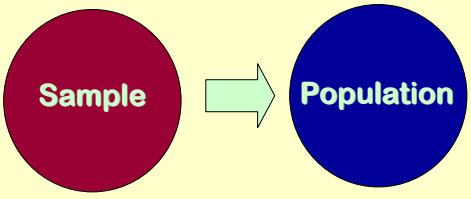
Psychological Statistics

Inferential Statistics: An Introduction

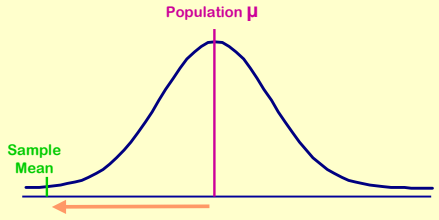
What We Will Cover in This Section

- Introduction.
- Probability and the Normal Curve.
- Probability and Sampling Means.
- The Z-test.

General Model



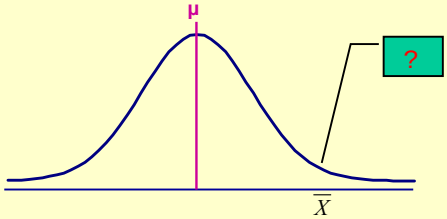
The Statistical Model



Probability and Sampling Means

Key Concept

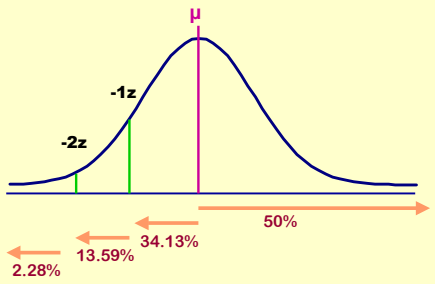
What is the probability that a sample mean came from a particular population?



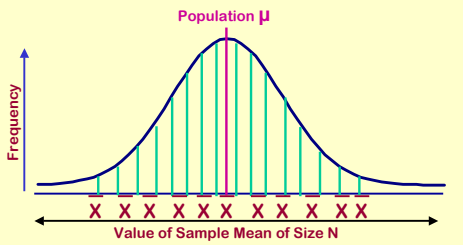
Issue

To what degree does a sample mean represent the population to which we want to make inferences?

Slicing the Normal Curve, Again

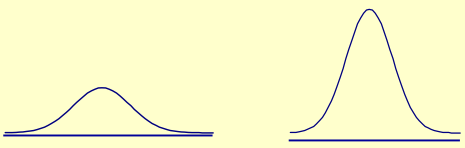


Distribution of Sample Means



Incredibly Cool Demonstration

Distribution of Sample Means



Small Sample Large Sample

Sampling Error

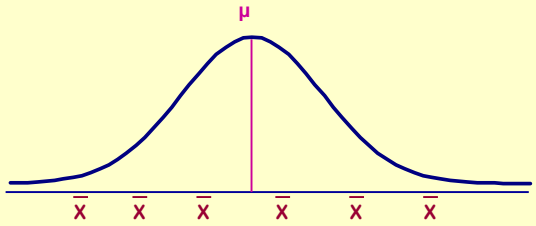
The degree to which a sample statistic deviates from its corresponding population parameter.

Sampling Distribution

The distribution of statistics of a given size (N) taken from a population.

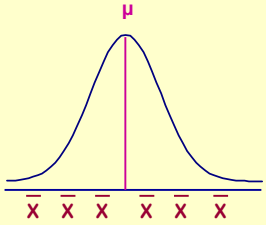
When the Sample Size is Small

N=10



When the Sample Size is Large

N=25



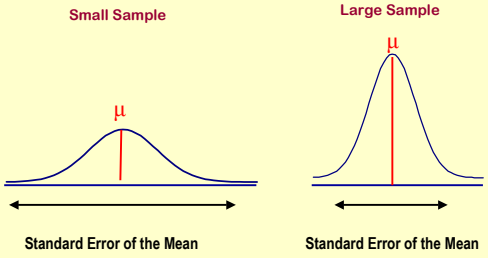
Central Limit Theorem

- When a large number of sample means of size N are selected from a population they will be normally distributed.

KEY POINT

- As the sample size gets larger, the standard deviation of the sample means gets smaller.
- The standard deviation of the sample means is called the STANDARD ERROR OF THE MEAN.

Standard Error of the Mean, $\sigma_{\bar{X}}$



Standard Error of the Mean

The standard deviation of the distribution of sample scores around the population mean.

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$$

Example

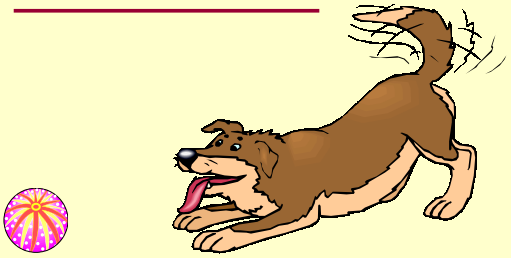
Given	Given
$\sigma = 24$	$\sigma = 24$
N = 64	N = 36
What is $\sigma_{x?}$	What is $\sigma_{x?}$

Key Learning Points

1. The normal distribution can be used to describe the distribution of many naturally occurring variables.
2. The CLT tells us that the distribution of sample means approximates the normal distribution.
3. The standard deviation divides the normal distribution into meaningful units.
4. We can describe the probability of randomly selecting any score from the normal distribution. All we need to know is how far that score is from the mean in standard deviation units.

Another Phenomenonally Cool Demonstration

A Tale of Tails



Research Predictions

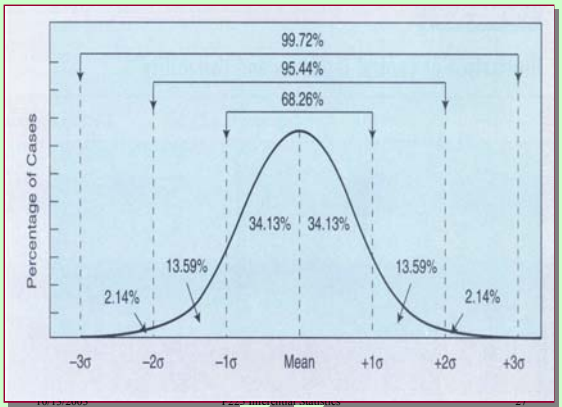
1. One can predict that one group is bigger than another.
2. You can predict that two groups differ but you don't know which will be bigger.

One-tail Prediction

- A research hypothesis where one group is predicted to be larger than another.
- Example.
  - Continuous reinforcement will lead to faster learning than intermittent reinforcement.
  - Babies who are held by their parents will have higher self esteem than babies who are not held by their parents.
  - Students who cram will have lower grades than students who don't cram.

Two-tail Prediction

- A research hypothesis where one group is predicted to be different than another but the researcher does not know if they will be higher or lower.
- Example.
  - Taking a nap before a meal will either increase or decrease your appetite.
  - Listening to music while taking a test will either help or hurt your grade.
  - Students who wear a bow tie to class will either impress or irritate their handsome instructor.



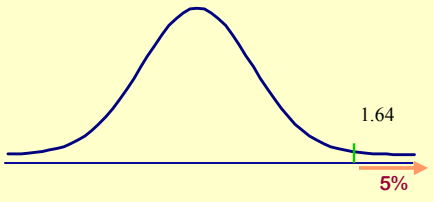
What Is Rare?

Something that would happen less than 5% of the time is 'rare' from the statistician's point of view.

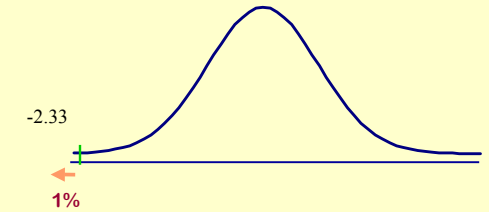
Key Probabilities (Critical Values)

Above what z-score do 5% of the cases fall?	1.64
Below what z-score does 1% of the cases fall?	2.33
Between which two z-scores do 95% of the cases fall?	"1.96
Between which two z-scores do 99% of the cases fall?	" 2.58

Above what z-score do 5% of the cases fall?

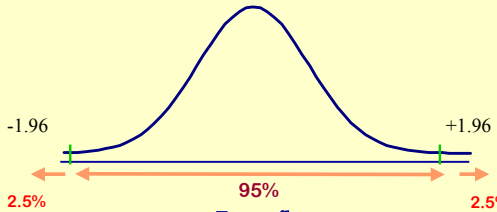


Below what z-score does 1% of the cases fall?



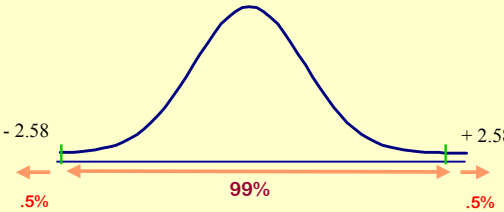
One tail

Between what two z-scores do 95% of the cases fall?



Two tails

Between what two z-scores do 99% of the cases fall?



Two tails

Critical Value

A numerical index that allows us to decide whether a statistical results is rare.

The Z-Test

Application: The Z-test

The average age of registered voters in Slippery Gulch is μ = 39.7 years old and the standard deviation, σ, is 10.

The League of Women Voters wanted to encourage younger people to vote so they sponsored a series of educational articles and television commercials on the benefits of voting.

Afterwards, a sample of 12 voters at the latest election was found to have a mean age of 28.2 years.

Did the advertising have an effect on voters or could this result have been a result of random error?

How to Think About This

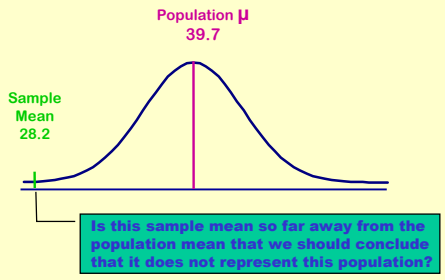
Could a sample with a mean of 28.2 have occurred in a distribution where the mean is 39.7 and the standard deviation is 10?

or

Does the sample with M = 28.2 represent a different population?

What distinguishes this 'different' population would be the commercials.

The Statistical Model, Again



How to Think About It

- How do you determine far away?
  - What measure do we have to determine how far away a sample mean is from the population mean?
- How do we determine if this mean is rare?
  - What is rare?

The Z-Test Formula

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

How to Compute, Step #1

1. Determine the critical value for a one-tail test where p < .05.

Critical Value = -1.64

How to Compute, Step #2

2. Calculate the standard error.

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$$

$$\sigma_{\overline{X}} = \frac{10}{\sqrt{12}}$$

$$\sigma_{\overline{X}} = \frac{10}{3.464}$$

$$\sigma_{\overline{X}} = 2.89$$

How to Compute, Step #3

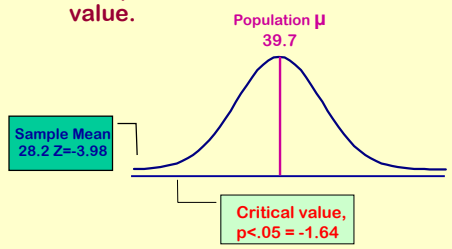
3. Calculate how far the sample mean is from the population mean in SE units.

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} \qquad Z = \frac{28.2 - 39.7}{2.89}$$

$$Z = -3.98$$

How to Compute, Step #4

4. Compare the Z-score to the critical value.



Properties of the Z-test

- What you can learn.
  - Does a sample mean (M) differ significantly from a population mean (:) or could this difference have occurred by chance.
- Assumptions.
  - Interval or ratio scales.
  - Know μ and σ.
  - Know the sample mean.
  - Know the sample size.

ALPHA Level (α)

- ***ALPHA*** **is the statistical statement of something that is rare.**
  - **Traditionally, *alpha* is defined as something that would happen 5% of the time or less.**
  - **This is shown by: p < .05.**

Critical Values for α

Critical Value	Type of test	
	One tail	Two tailed
.05	1.64	1.96
.01	2.33	2.58

Example #2

Melody Tunne thought that listening to music while taking a statistics test would either be relaxing, increasing performance, or distracting, decreasing performance. She did not know which.

1. **Is this a one-tail or two-tail test?**
2. **What alpha level should Melody set?**

Melody's Data

- The mean for the population of students who have taken the statistics test is $\mu = 50$.
- The standard deviation for all students is $\sigma = 12$.
- Melody got a sample of 49 students who listened to music while taking the test.
  - Their mean was 54.63
  - Their standard deviation was 7.